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ABSTRACT

This document provides abstracts of 20 research reports. Topics covered include: children's comprehension of simple story problems; field independence and group instruction; problem-solving competence and memory; spatial visualization and the use of manipulative materials; effects of games on mathematical skills; problem-solving ability and right hemisphere processing; effects of calculator use; effects of spatial instruction; conceptualization of subtraction; cognitive processes used in problem-solving; variables discriminating good and poor problem solvers; heuristic hints and the solution processes. (MP)

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MATHEMATICS EDUCATION REPORTS

Boston, Massachusetts
18-21 April 1979

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RESEARCH REPORTING SECTIONS

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

57TH ANNUAL MEETING

Jon L. Higgins, Editor

The ERIC Clearinghouse for Science, Mathematics
and Environmental Education
The Ohio State University
College of Education
1200 Chambers Road, Third Floor
Columbus, Ohio 43212

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MATHEMATICS EDUCATION REPORTS

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FOREWORD

This Mathematics Education Report contains abstracts of all papers presented in the research reporting sessions of the 57th Annual Meeting of the National Council of Teachers of Mathematics. These papers were selected for presentation from a large number of proposals submitted to the NCTM Research Advisory Committee and the program chairman for Research Sections, Steve Krulik. We wish to thank Dr. Krulik and the members of the Research Advisory Committee for making them available for this publication.

Jon L. Higgins
Associate Director for
Mathematics Education

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Reporting Session

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10:30-11:30

AN INVESTIGATION OF FACTORS ASSOCIATED WITH CHILDREN'S COMPREHENSION OF SIMPLE STORY PROBLEMS INVOLVING ADDITION AND SUBTRACTION PRIOR TO FORMAL INSTRUCTION ON THESE OPERATIONS

Cheryl Gibbons Ibarra and C. Mauritz Lindvall
University of Pittsburgh

Purpose

The purpose of this study was to investigate the extent to which kindergarten pupils, prior to any formal instruction in addition and subtraction, can solve simple story problems involving the union of sets or removal of subsets, merely through counting. The study also attempted to determine the extent to which this type of performance varied with problem complexity and with the extent to which manipulative aids were used in presenting the story.

A significant portion of elementary school mathematics focuses on the representation of arithmetic problems through the use of number sentences and such related tasks as the solution of open number sentences. This symbolic representation of addition and subtraction is typically introduced to primary grade pupils by having the teacher describe an appropriate story problem and then show the number sentence that represents the story. However, comprehension of such number sentences requires that the children have the ability to understand the physical concepts and operations which the symbols represent. That is, comprehension of number sentences can be considered to be a reading comprehension task, and students must have the "reading readiness" represented by an understanding of the physical conditions and actions that the number sentences describe before they are ready to comprehend the sentences. Prior to formal instruction young children appear to be capable of solving addition and subtraction problems when they are presented in a simple, non-symbolic form (Buckingham and MacLatchy, 1930; Gelman, 1977; Brush, 1978). However, past work in this area has not attempted to determine the extent to which this capability is dependent upon specific characteristics of the stories involved or upon method of presentation. The present study is an effort to contribute to this need. Specifically, it investigates (1) the effect of certain aspects of story complexity, and of (2) degree of concreteness used in presentation of the story to the child upon children's ability to solve simple oral stories through the use of counting.

Procedure

Approximately 120 kindergarten children from five different schools located in five different types of communities were interviewed. The

children were representative of a broad range of academic abilities and home and community environments and had received no formal instruction in addition and subtraction. The testing was conducted by the researchers on a one-to-one basis and required approximately 30 minutes per child. Each child was given the opportunity to respond to 30 story problems which were presented orally and which varied in the complexity of the problem and in the extent to which manipulative materials were used in the presentation.

Design

The five variations used in the presentation of the story were as follows:

1. Story Only. Problem read to child with no manipulatives provided.

2. Objects Available. Manipulative pieces representing the objects in the story were available to the child as the story was read.

3. Sets Shown. Physical representations of the sets of objects described in the story (but no physical operations) were provided as the story was read.

4. Pictured Description. Pictorial representations of the sets of objects and of the physical operations described in the story were provided as the story was read.

5. Sets and Actions Shown. Physical representations of the sets of objects and the physical operations described in the story were provided by the tester as the story was read.

These variations in degree of concreteness were used in the presentation of six problem types which varied according to the operation, the position of the unknown, and whether a transformation or static situation was described.

Analysis

A repeated measures factorial design was used in order to test all children on each of the six types of problems administered under each of the five manipulative conditions. This permitted the investigation of the differences associated with problem types, differences associated with method of problem presentation, and interactions. For each cell the proportion of students showing mastery of that type of problem under the specified conditions of administration was determined. Also, the marginal proportions were determined in order to examine the main effects of the two major variables.

Results

Results of the study are summarized in Table 1. As was anticipated, the analysis of the marginal differences associated with method of problem presentation showed that method 1, consisting of having the story read with no manipulatives available, was the most difficult and method 5, consisting of a rather complete demonstration of set sizes and of the physical operations described in the story, was the easiest. The other three methods of presentation were of intermediate difficulty and did not differ significantly from one another. The order of difficulty for problem types can be seen by studying the proportions in the right-hand margin of Table 1.

Table 1. Proportion of Children (N = 118) Responding Correctly to Oral Story Problems of Specified Complexity When Presented Under Varying Degrees of Concrete Representation

		Story				
Story Type	Story Only	Objects Available	Sets Shown	Pictured Description	Sets and Actions Shown	All Conditions
a + b = □ Transform.	.42	.54	.68	.58	.75	.60
a + b = □ Static	.48	.55	.79	.69	.79	.66
a - b = □ Transform.	.28	.35	.34	.39	.97	.46
a + □ = c Transform.	.06	.17	.15	.32	.50	.24
a + □ = c Static	.14	.25	.13	.29	.68	.29
a - □ = c Transform.	.31	.50	.27	.47	.84	.48
All Stories	.28	.39	.39	.46	.75	

An examination of the proportions in the individual cells shows that, with one exception, the order of the magnitude of the proportions within each column and each row are not significantly different from that found in the two margins. This suggests that there is no simple interaction between the two major variables of this study.

Conclusions

Results of this study suggest that a majority of kindergarten pupils, prior to formal instruction in addition and subtraction, comprehend the types of stories frequently used in the later introduction of these operations and are able to solve such stories merely by counting.

It also indicates that the chances that the problem will be understood can be enhanced if maximum use is made of manipulative materials in presenting the story. However, the study also shows that a definite proportion of children are unable to comprehend even the simplest stories when presented under optimal conditions. This suggests that some diagnosis of the extent to which this is true in any given classroom would provide important information for meeting the problems of students who have difficulty in beginning arithmetic.

Reporting Session

Thursday, April 19
10:30-11:30

THE INTERACTION OF FIELD INDEPENDENCE WITH SMALL GROUP INSTRUCTION IN MATHEMATICS

Douglas B. McLeod and Verna M. Adams
San Diego State University

Purpose

Research on mathematics teaching indicates that no one instructional treatment is likely to maximize learning for all students. Instead of looking for a single treatment that will be best for all learners, Cronbach (1957) has suggested that different instructional strategies should be used for students with different characteristics. Attempts to adapt instructional treatments to student characteristics are known as Aptitude-Treatment-Interaction (ATI) studies.

A complete review of ATI research has recently been completed by Cronbach and Snow (1977). Although there have been many difficulties in conducting ATI studies, Cronbach and Snow conclude that interactions do exist and that ATI research has important implications for individualizing instruction.

Recently several ATI studies in mathematics have found interactions using the cognitive-style variable of field independence (Horak and Zweng, Note 1; McLeod and Adams, Note 2; McLeod, Carpenter, McCornack and Skvarcius, 1978). Field independence is a rather stable trait that is related to both the performance of cognitive tasks and to personality characteristics. Field-independent students tend to do well at tasks that require restructuring or reorganization of the content, especially if the content deals with mathematics or science; they also seem to have more personal autonomy than field-dependent students. Field-dependent students, on the other hand, are more adept in social situations where they seem to exhibit greater interpersonal skills. However, they have more difficulty with tasks that require restructuring ability (Witkin and Goodenough, Note 3; Witkin, Moore, Goodenough and Cox, 1977).

These differences between field-independent and field-dependent students seem to be related to performance in discovery-oriented instruction. For example, McLeod et al. (1978) found a significant interaction between field independence and the level of guidance of instruction. Field-independent students excelled in the treatment that provided minimal guidance and maximal opportunity for discovery. Field-dependent students, however, learned more in an expository treatment where lots of structure was provided. A later study found a similar interaction (McLeod and Adams, Note 2). Horak and Zweng (Note 1) found a somewhat different interaction. In their study, field-dependent students achieved most in the discovery-oriented

treatment, probably because that group allowed for more student interaction and social contact. Field-independent students tended to perform at about the same level whether they were in the inductive, discovery treatment or the deductive, expository treatment.

The purpose of this study was to investigate further the effect of social interaction on the learning of field-dependent students in inductive instruction. Treatments differed in their use of small-group as opposed to independent work. The hypothesis to be tested was that these treatments would interact with measures of field independence. Field-dependent students were expected to do better in small-group instruction, while field-independent students might be held back by small-group work. However, field-independent students should do better working independently.

Procedure

The participants in the study came from five sections of a mathematics class for prospective elementary school teachers. Within each section, subjects were randomly assigned to either small-group or independent instruction.

Both treatment groups studied the same unit on the topic of networks. The unit presented a variety of concepts, including equivalence of networks, traversability and its applications, Euler's formula, and related topics. The unit encouraged students to make discoveries through its use of an inductive sequence of instruction, presenting first a number of examples from which students could then generalize to obtain a rule. The only difference between the two treatment groups was that in one the students were assigned to work in groups of four, and in the other the students were asked to work independently. The small-group treatment was conducted in the mathematics laboratory, while the students who worked independently stayed in the regular classroom. The two instructors, who were randomly assigned to treatments, answered questions by encouraging students to look for patterns and to make discoveries.

Students were assigned to small groups according to their scores on a measure of field independence. Each small group contained one student from the lowest quartile and one from the highest quartile, along with two students from the middle 50 percent of the scores.

Students were assessed on field independence using the Hidden Figures Test and the Group Embedded Figures Test. SAT scores were used as a measure of general ability. Following the period of instruction, students were assessed on immediate achievement; the same test was readministered eight weeks later as a measure of retention. Students were also asked to evaluate the quality of the unit.

Two regular 75-minute class periods were used by the students to complete the treatments and to take the posttest. Most students were not able to complete the material in the time allowed.

Results

Analysis of the data is now underway using multiple regression procedures. Preliminary results indicate that there is an interaction with Hidden Figures Test scores for immediate achievement, but not for the retention test. Also, there appears to be a significant difference between the treatment groups in their evaluation of the unit.

Reference Notes

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Reporting Session

Thursday, April 19
1:00-2:00

PROBLEM-SOLVING COMPETENCE AND MEMORY FOR MATHEMATICAL PROBLEMS: CUE SALIENCE AND RECALL

Edward Silver
Northern Illinois University

Purpose

The major purpose of this study was the examination of the relationship between a student's problem-solving performance and his or her memory for mathematical problem information. The investigation also sought to clarify the relationship between a student's initial perception of salient cues in a problem statement and his or her subsequent recall of information about the problem.

Conceptual Framework

When solving a new problem, a successful problem solver presumably uses information, procedures, and more general notions that have been obtained from previous experience and training. Thus, a person's mathematical problem-solving competence may be intimately related to the ability to extract important information from a given problem solution episode and encode the information for later retrieval and the ability to recall successfully the important features of the previous problem-solving episode and apply them to the solution of the present problem.

Recently published research by Krutetskii (1976) has suggested that good and poor problem solvers differ in their recall of information from previously encountered problems. In particular, good problem solvers apparently tend to recall the structural features of a problem, whereas poor problem solvers tend to recall, if anything, the specific details of a problem statement.

The work of Chartoff (1976) and Silver (1977) has suggested that students may also differ in their perceptions of problem similarity. Silver identified four dimensions along which students viewed problem relatedness: mathematical structure, contextual details, question form, and pseudostructure. The first three are self-explanatory; the latter refers to problem similarity based on the presence of a common measurable quantity, such as age or weight. Silver's data indicated that the perceived salience of the mathematical structure dimension was significantly related to problem-solving competence. Thus, his research suggests that good and poor problem solvers may differ in their encoding of problem information, since they differ in their perceptions of cue salience.

To examine adequately the nature of students' memories for mathematical problems, it would appear necessary to consider the students' encoding behaviors as well as the nature of their recall. Recent research by Bjorklund, Ornstein and Haig (1977) has suggested that input (i.e., encoding) organization is a mediating factor in memory performance and development. Furthermore, Mandler and Johnson (1977) have reported that the schemata used by subjects to guide encoding of a story are related to, but not identical to, those used to guide retrieval of story information.

An unpublished exploratory study by Kilpatrick and Silver conducted during the 1974-75 academic year examined Krutetskii's hypothesis and explored alternative means of measuring memory for problems. Their work has suggested the feasibility of using paper and pencil, group-administered instruments in measuring students' recall of problem information. Their findings indicated support for Krutetskii's suggestion that good problem solvers recall problems differently than do poor problem solvers, but the findings also suggested that the relationship may be artifactual since poor problem solvers may never be aware of the problem structure.

The present study extended the exploratory work of Kilpatrick and Silver by examining subjects' recall of problem information along each of the four dimensions identified by Silver (1977), by comparing the cue salience evident in a subject's pre-solution and post-solution judgments of problem relatedness with the dominant features in his or her recall of problem information, and by further examining the relationship between problem-solving ability and memory for problems by assuring that all students are aware of a problem solution before being asked to recall information about the problem and by investigating the nature of a subject's recall of previously solved problems in new problem-solving situations.

Procedure

The investigator constructed a Card-Sorting Task (CST) similar to the one used by Silver (1977) in which 16 verbal problems were each placed on a separate card. The problems were written so that they varied systematically along the four dimensions identified by Silver. The problem set was constructed so that a person attending to only one of the dimensions would partition the set into four disjoint subsets of four problems; each resulting subset would contain problems that were related only along the given dimension.

A measure of a student's problem-solving competence was obtained by having the students solve each of the CST problems. Before and after the problem solutions, students were instructed to remember everything that was "mathematically important" about the problems.

Three problem memory tasks were constructed. The first was simply a free recall task, in which subjects were instructed to "write down everything that you can recall about each of the problems" from the CST set. No clues were present in their response booklet, nor were they given orally.

The second memory task involved cued recall of the CST problems. For each of the four dimensions, two clues were developed to prompt recall of a certain CST problem or a subset of CST problems.

The third memory task was the Related Problems Test (RPT), which consisted of four problems similar to, but not identical to, problems from the CST. Each RPT problem was related along at least two dimensions to subsets of CST problems. Administration of the RPT proceeded as follows: (1) students were told to read the first RPT problem, to think about how they would solve it, and to write down as much information as they could recall about CST problems that they judged to be "mathematically related" to the given problem, then they applied this procedure for the remaining three RPT problems; (2) students were instructed to solve each of the RPT problems and to record their work; and (3) after solutions were collected, students were given the RPT problems again and asked to write down as much information as they could recall about CST problems that they now judged to be "mathematically related" to the given problem.

The sample consisted of 56 eighth-grade students enrolled in non-algebra mathematics classes. Students completed the CST on two occasions, once before and once after solving and discussing the solutions to the CST problems. On the day following the first CST sort, students solved problems 1-8; the solutions to problems 1-8 were discussed on the day following the attempted solutions. The same procedure was followed for problems 9-16.

The three memory tasks were administered on successive days following the second CST sort. The free recall task was given first, followed by the cued recall and RPT tasks respectively.

Results

The data have been collected but analysis has not yet begun. At the time of the meeting a complete report of the results will be given.

The free recall data will be examined using a modification of the scheme developed by Kilpatrick and Silver. The CST data will be quantified using the scoring system developed by Silver.

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Reporting Session

Thursday, April 19
1:00-2:00

THE INTERACTION BETWEEN SPATIAL VISUALIZATION ABILITIES AND THE USE OF MANIPULATIVE MATERIALS IN THE ENACTIVE AND ICONIC MODES FOR PRESERVICE ELEMENTARY TEACHERS

Phillip M. Eastman
Boise State University

and

Jeffrey C. Barnett
Northern Illinois University

Purpose

The use of manipulative materials for teaching mathematics in the elementary grades has received considerable support from many components of the mathematics education community. Despite this growing interest in the use of these materials, the amount of research on the preparation of teachers to use manipulative aids is quite limited. In the absence of directives from empirical research, decisions concerning the role of manipulatives in the elementary mathematics methods course have been made on practitioners' "hunches" or on assumptions of the effectiveness of the use of such materials. In view of the expense of manipulative aids and the relatively large amount of class time needed to provide prospective elementary teachers "hands-on" experiences with these materials, the lack of an empirical basis for this decision-making process is unacceptable.

In 1976, Barnett and Eastman conducted a study to determine if prospective elementary teachers need actual "hands-on" experience with manipulative aids to be able to demonstrate numerical and structural properties of the four basic arithmetic operations. The results showed no significant difference between the "materials" group and the "non-materials" group on measures of performance in either the enactive mode (concrete) or the iconic mode (pictorial representation). Significant differences favoring the "materials" group were found on a measure of mathematics achievement.

The purpose of the present study was to:

1. Extend the 1976 study to include instructional material on more difficult and complex algorithms.

2. Investigate the possible interaction between "material" and "non-material" treatments, with measures of visual ability.

3. Assess the effects of the use or non-use of concrete materials upon attitude (Aiken, 1976).

Procedure

The subjects for the study consisted of 73 elementary education majors assigned to two sections of an elementary mathematics methods course. The subjects were randomly assigned to either an experimental or a control group. Both groups met together twice each week for a one-period lecture-discussion session, and once each week separately for a double period laboratory session. The experiment was conducted over a seven-week period. The experimental ("materials") group was assigned to the mathematics education laboratory, while the control ("non-materials") group was assigned to a regular classroom in the same building at the same time.

The topics for instruction consisted of seven laboratory units on properties and algorithms for the four basic arithmetic operations taken from a laboratory manual for preservice elementary teachers (Jungst, 1975). Each unit consisted of several demonstrations and examples, followed by a set of exercises which required the students to draw pictures of the manipulative devices to demonstrate the arithmetic properties (associativity, commutativity, etc.) and algorithms. The experimental group was required to demonstrate the arithmetic properties and algorithms on their desks, using the concrete materials, before they were permitted to complete each exercise in the laboratory manual. The control group was not provided with concrete materials and worked the laboratory exercises in the manual entirely in the iconic mode.

Following the laboratory sessions in the third and seventh weeks, unit tests were administered to all subjects. The laboratory tests required responses in the iconic mode and were similar to the exercises found in the laboratory manual. A mathematics concepts test was also administered after the third laboratory session to assess arithmetic achievement. To assess student ability to physically demonstrate properties and algorithms using designated manipulative aids, 40 subjects were randomly selected to be interviewed by the investigators. Each of these students was presented two properties and two algorithms and were required to demonstrate them using centimeter rods, pegboards and set demonstration objects. Each demonstration was scored according to a point scale which gave partial credit for each correct step.

Analysis

The study employed a randomized posttest design. The hypotheses were tested using a linear regression model. The alpha level of rejection was .05. Pre and posttest attitude measures were taken using Aiken's scale.

Results

The results of this study were consistent with the results from the previous Barnett and Eastman study. In no instance did the experimental ("hands-on") group perform better than the control group,

either on paper scored tests or the interviews where both groups were asked to demonstrate ability to manipulate materials. Neither were there any interactions between the mode of instruction and the visualization ability of the subjects. There were no significant attitude changes during the experiment for either the experimental or control group.

Conclusions

This experiment confirms the previous result that when working with the basic properties of the whole number system, it is not beneficial to ask preservice elementary teachers to actually use manipulative materials. Indeed, in the previous study and in this study students who only saw demonstrations with the manipulatives did as well as those who were asked to actually use manipulatives in working problems; and the subjects who only watched the demonstrations were able to finish their work in a much shorter time period than those who were required to manipulate the materials. This result held when the learning material was the more complex task of learning to teach algorithms as well as when the learning task was properties of the whole number system. Most significantly, there were no differences between the groups in the ability to demonstrate how the manipulatives could be used to demonstrate arithmetic properties and algorithms. The authors' suggestion that there might be an interaction between the mode of learning the material and the spatial visualization ability of the subjects gains no support from the results. In retrospect, we suspect that the visualization ability which is required to learn from the illustrations is too low to be outside the range of most subjects. Hence, differential visualization ability is probably not a primary consideration in determining the most effective use of manipulative aids. The extension of the experimental time period to seven weeks and the inclusion of the more complex learning material of algorithms for addition, subtraction, multiplication, and division strengthen the results of this study. Based on these results and the results of the Barnett and Eastman study, it is apparent that the cost of instruction using a "hands-on" approach to the teaching of the use of manipulative materials is unwarranted. Indeed we should not be rushing into the process of equipping lavish mathematics laboratories for instructing preservice elementary teachers in the art of teaching mathematics to elementary school children. Rather we should instruct them in the use of manipulative materials by demonstration and through the use of pictures of the materials. This appears to work as well in helping them learn both the methodology of teaching the basic properties and algorithms for whole numbers and the mathematics content itself. The latter method also has the benefits of economy of time and lower instructional costs which are of prime consideration in this period of cutbacks in resources for education.

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EFFECTS OF GAMES ON MAINTENANCE OF MATHEMATICAL SKILL AND ON QUANTITY AND QUALITY OF MATHEMATICAL VERBALIZATION

George W. Bright
Northern Illinois University

John G. Harvey
University of Wisconsin - Madison

Margariete Montague Wheeler
Northern Illinois University

It is averred that games are appropriate instructional activities. There exists, however, a paucity of empirical research about the cognitive effects of games used in mathematics instruction (Bright, Harvey and Wheeler, 1977). Some cognitive effects of games have been recently investigated (Bright, Harvey and Wheeler, 1978) but only at the post-instructional level. In order to examine further the model for research on games of Bright, Harvey and Wheeler (1977), three studies were planned to use games at the pre-, co-, and post-instructional levels. One study was an extension of the previously cited work, and the other two studies investigated the effects of games on students' verbalizations. Verbalization is part of the learner-learner-interaction dimension of the previously cited model.

Study 1: The effect of games on the maintenance of skill with basic multiplication facts

Purpose: The study addressed the question, "Can games be used to maintain mastery of basic facts?" That is, once basic facts have been mastered, how often should these facts be practiced in order that mastery be maintained?

Game: The game used in this study was MULTIG from Developing Mathematical Processes (DMP) (Romberg, Harvey, Moser and Montgomery, 1974, 1975, 1976), a basic fact multiplication game similar to CONTIG (Broadbent, 1972). The game was used at the post-instructional level.

Procedure: Approximately 100 students enrolled in four grade 5-6 classroom units in MacFarland, Wisconsin, were the subjects. At the beginning of the experiment (20 September 1977) the interval between the 15-minute game-playing sessions was six instructional days. Two instructional days after each game-playing session, a two-minute speed test of 20 randomly selected basic multiplication facts was administered. For each class, scores on successive speed tests were subjected to a t-test. If there was a statistically significant drop ($p < .05$), the interval between playing sessions was decreased by one instructional day. When a decrease was not detected, the interval was

increased. In late May the interval between game-playing sessions was 25 instructional days. Between September 1977 and May 1978, the game was played 12 times. Five weeks after the last game-playing session, a five-minute, 100-item speed test of all basic multiplication facts was administered. This same test was used as the pretest.

Study 2: The effect of games on mathematical verbalization

Purpose: The first phase of the study was to examine whether the amount of mathematical verbalization is affected by altering the constraints imposed upon the players; e.g., amount of recordkeeping. The second phase was to determine whether the constraints which were associated with increased verbalization were also associated with increased achievement.

Games: A variety of games from DMP were used. The content included fractions, whole number division, metric measurement, geometric shapes, and multiplication. The games were used at the pre-, co-, and post-instructional levels.

Procedure: The first phase was conducted in winter 1978 with small groups (N = 4) of fourth-grade students from a single class over a period of five weeks in Rochelle, Illinois. All sessions were videotaped. Each group played either a single game repeatedly with the constraints changed after each game or several different games with different constraints. The constraints that were changed included the following: amount of recordkeeping, playing as teams versus playing as individuals, use of calculator versus paper-and-pencil or mental calculations, different scoring procedures, levels of memory load, and different devices for generating information.

The second phase was conducted with two fourth-grade and two fifth-grade classes (N = 20, approximately, in each case) in a different school in Rochelle. In each class, students were ordered on previous mathematics achievement and were grouped by pairs; one member of each pair was randomly assigned to one of the two treatments for that class. In each of these four substudies one of three independent variables was manipulated: amount of information recorded, incorporation of instructional objective into the rules, or device for generating numbers for use in computations. Each treatment lasted 12 days: day 1, pretest; days 2 through 11, game-playing; day 12, posttest.

Study 3: The effect of games on mathematical description

Purpose: The study investigated the effect of games in increasing a young child's ability to describe mathematical shapes in accurate and meaningful ways. In addition, the study examined the effect of playing mathematical games on the numerical or verbal (oral) proficiency of the player.

Game: Two games similar to Stacking as found in DMP were used. The instructional objectives for each of the two-person games were (1) given a two-dimensional shape, the child describes the shape, and (2) given a description, the child selects a shape which matches the description. The games were identical except for the game pieces: plastic geometric pieces (e.g., triangles and hexagons) or colored tracings of these pieces on blank playing cards.

Procedure: Students who will enter kindergarten Fall 1978 were subjects for the study. The students were enrolled in private preschool programs in DeKalb, Illinois, Summer 1978. The treatments were conducted during a ten-day period. On day 1 each student became familiar with the game pieces and took a verbal and a numerical pretest. On days 2 through 8, students played the game each day with the same partner. On day 9, the numerical and verbal proficiency instruments were administered as posttests. In addition, the test for meaningfulness of description was administered and the students became familiar with the three-dimensional solids to be used in the transfer set. The transfer test was administered one week later.

Analysis

The data have been collected for study 1 and study 2. The data will be collected for study 3 before 1 August 1978. Results for study 1 provide support for the research question. Results for study 2 and study 3 will be given at the reporting session.

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Reporting Session

Thursday, April 19
4:00-5:00

INTERRELATIONSHIPS BETWEEN PROBLEM SOLVING ABILITY, RIGHT HEMISPHERE PROCESSING FACILITY, AND SUCCESS IN MATHEMATICS INSTRUCTION

Michael Battista
University of Central Arkansas

Purpose

Currently there is much research being done in mathematics education which has as its goal the increasing of our understanding of how students solve problems. Important topics to be considered in problem-solving research are the mental processes occurring in students as they attempt to solve problems, the abilities students utilize in solving problems, the methods by which teachers can improve their students' problem-solving capabilities, and the role of problem-solving ability in the learning of mathematics.

Another area of research that is currently gaining interest among mathematics educators is the hemispheric specialization of the brain. The two hemispheres of the brain are specialized for different modes of thought processing (Wheatley, 1977). In normal persons the left hemisphere excels in performing sequential tasks, logical reasoning, and analysis of the components of a stimulus. Language is processed in the left hemisphere. The right hemisphere, on the other hand, processes stimuli all at once rather than sequentially; it senses stimuli holistically. The right hemisphere excels in spatial tasks and tasks that are nonverbal in nature.

Up to the present time there has been little, if any, interaction between problem-solving research and hemispheric specialization research. It seems likely, however, that these two areas of research can be profitably interrelated. In fact, it has been suggested by some researchers in hemispheric specialization that the interplay between right and left hemisphere processing is a factor that should be considered in attempts to understand the processes involved in problem solving (Wheatley, et al., 1978). Furthermore, both of these areas of research should have important implications for theories of how students learn mathematics.

The present research effort represents an initial attempt to relate problem solving, hemispheric specialization, and mathematics learning. More specifically, the purpose of the study was to investigate the interrelationships between problem-solving ability, right hemisphere processing facility as evidenced by performance on a spatial visualization task, and grade performance in college mathematics courses.

Procedure

The subjects of the study were students at the University of Central Arkansas who were enrolled in College Geometry (CG), Geometry for Elementary and Middle School Teachers (EG), or College Algebra (CA). The students in CG were mathematics majors who had all been through the calculus sequence, the students in EG were elementary education majors, and the students in CA represented the usual variety of majors enrolled in an introductory college algebra course. CG used an axiomatic approach in covering Euclidean geometry and some finite geometry. EG covered informal intuitive geometry using an activity-oriented approach.

At the end of each semester, each student was given an experimenter-constructed problem-solving test (P) consisting of nonroutine problems and the Purdue Spatial Visualization Test: Rotations (S) [Guay, 1977]. In addition, each student received a numerical course grade score of 15, 14, 13, . . . (denoted G) which corresponded to a letter grade of A+, A, A-, . . ., respectively.

The Purdue Spatial Visualization Test: Rotations is a 30-item group test designed to measure the ability to mentally rotate three-dimensional objects depicted in drawings. The tasks required by the test have been shown by EEG investigations to be right hemisphere activities (Wheatley, 1978). A high score on the test indicates high ability to utilize right hemisphere processing.

For each pair of variables S x P, P x G, S x G, the Pearson product-moment correlation coefficient was calculated; first for all three courses combined and second for each individual course. Examination was made of the intercorrelations between the three variables and of the role of instruction in the P x G and S x G correlations. Because of the nature of the instruction in the courses, it was predicted in particular that the S x G correlation would be highest for EG, next highest for CG, and lowest for CA.

In addition to the above correlations, the multiple linear regression, $R_{G.PS}$, of course grade on problem-solving ability and spatial visualization ability was calculated. Finally, the S x P, P x G, and S x G correlations were compared for males versus females, and differences between males' and females' S, G, and P scores were examined.

Results

The results of all the correlations are shown in Table 1. None of the t-test comparisons of male versus female scores were significant. The multiple linear regression of course grade on problem-solving ability and spatial visualization ability was $R_{G.PS} = .534$, significant at the .01 level ($F(2,67) = 13.367$).

Table 1

Intercorrelations Between Problem-Solving Ability (P), Spatial Visualization Ability (S), and Course Grade (G)

Group	Variables Correlated[a] S x P		P x G		S x G	
All	.467***	(70)	.519***	(76)	.353**	(74)
CG	.031	(20)	.364*	(23)	.325	(20)
EG	.564**	(21)	.649***	(23)	.577**	(25)
CA	.449**	(29)	.488**	(31)	.268	(29)
Males not in EG	.570**	(23)	.272	(24)	.388*	(21)
Females not in EG	.456***	(47)	.607***	(30)	.376*	(28)

[a]Numbers in parentheses indicate the numbers of pairs of scores used to compute the coefficients.

* $p < .05$.

** $p < .01$.

*** $p < .001$.

Conclusions

The correlation results observed in this study strongly suggest that there is a positive relationship between problem-solving ability and right hemisphere processing facility, and that both of these abilities are positively related to success in college level mathematics instruction. Furthermore, comparison of the S x G correlations for the three different courses suggests that right hemisphere processing facility is an important factor in activity-oriented geometry courses, while not so important in algebra courses and courses depending predominantly on logical reasoning abilities. The results seem to indicate that further investigation of the role that right hemisphere processing plays in mathematics learning and problem solving has a high potential for being fruitful. One conjecture that seems particularly worthy of further research is that improving right hemisphere processing facility through training will improve problem-solving ability and success in mathematics instruction.

Directions for further research are also suggested by some unexpected results appearing in the intercorrelations. First, the S x P correlation was much lower for the CG course than for the other two courses. This result could be an artifact of the testing instruments, or there might be some underlying cause for the observation. For instance, the students in CG were of significantly higher mathematical ability and had considerably more mathematical experience than the students in the

other two courses. It is possible that the S x P correlation is lower for the CG students because their previous mathematical experiences have forced them to develop left hemisphere strategies to compensate for weaknesses in right hemisphere processing (or vice versa).

The second unexpected result is the difference in P x G correlations for males and females. If this difference can be verified through further experimentation, it may indicate some basic difference between the strategies males and females use to learn mathematics.

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THE EFFECT OF CALCULATOR USE ON THE PROBLEM SOLVING STRATEGIES OF ELEMENTARY SCHOOL PUPILS

Charlotte L. Wheatley
Purdue University

Dramatic sales and low prices have stimulated many professional groups to call for experimentation with the use of calculators (NCTM, 1974, 1977; Higgins, 1974; NACOME, 1975; NIE-NSF, 1977). Studies of calculator use have generally taken the form of having one group of pupils use calculators for a fixed period of time while a second group studies the same material without a calculator. While this type of research is essential and valuable, there is a need to determine the changes in thinking strategies that may result from calculator use.

Problem solving in mathematics has three identifiable components. First, a decision must be made on how to solve the problem. This phase of the problem may employ a wide range of strategies ranging from applying a rote procedure to trial and error. Second, computations are performed. Third, the answer is evaluated. Persons may be distracted from learning or applying effective problem-solving strategies by performing the computations required. An elementary school pupil may spend several minutes dividing 6076 by 28. If this computation could be performed on a calculator, then he might be able to focus on strategies. Furthermore, different strategies might be feasible with a calculator. For example, iterative procedures may be reasonable with the possibility of performing many computations quickly.

By interviewing pupils as they solve problems it may be possible to gain insights into the impact of calculators on the problem-solving process.

In twelve empirical studies with elementary school pupils using calculators, nine reported results favoring the calculator group, one favoring the no calculator group and two reported mixed results (Allen, 1976; Borden, 1977; Campbell, 1976; Hawthorne, 1976; Jones, 1976; Nelson, 1976; Schafer, 1975; Schnur, 1976; Spencer, 1975; Sutherlin, 1977; Vanatta and Hutton, 1977; White and Shumway, 1977). In five of these studies (Campbell, 1976; Hawthorn, 1975; Jones, 1976; Spencer, 1975; White and Shumway, 1977) the differences were on tests of concepts, reasoning, or problem solving. Thus, evidence exists that calculators can facilitate thinking in mathematics with little risk of loss in computational proficiency.

To date no study has been found that attempted to examine the problem-solving strategies of persons using calculators. Weaver and Moser (1977) provide a rationale for such a study.

> Because calculators compute instantly, one of the benefits that Weaver and Moser see in them is that they can save the time needlessly spent on adding and subtracting and give children more time for learning how to think through problem solving (p. 4).

Purpose

With the availability of calculators, mathematics problem-solving strategies of elementary school pupils may be altered. When solving a problem, it is often necessary to perform difficult computations. The act of switching into the computational mode may detract from the development of effective problem-solving heuristics. Furthermore, with calculator availability pupils may utilize different problem-solving strategies. The purpose of this study was to compare the problem-solving strategies of elementary school pupils with and without the use of calculators. The following research questions were addressed:

1. Do elementary school pupils use a wider range of problem-solving strategies with a calculator than pupils not using a calculator?

2. Do pupils tend to use different strategies when a calculator is used in problem solving?

3. Do elementary school pupils solve more problems correctly when using a calculator?

4. Is there a difference in solution time when calculators are used?

Procedure

The sample for this study consisted of 50 fifth-grade pupils from a rural midwestern school with average achievement scores. Twenty-five of these pupils were from a class that had been using calculators in mathematics class for three months prior to the study. The other 25 pupils were randomly selected from two classes (56 pupils) with no previous in-class calculator experience. The class using calculators had been randomly selected from the four comparable fifth-grade classes at that school as part of another research project.

Over a six-week period, numerous mathematics word problems were piloted with pupils from a different elementary school. Through analysis of the pupil responses, five problems were selected for use in this study. Problems were selected using the following criteria:

1. The problems can be understood by fifth-grade pupils.
2. The problems can be solved in more than one way.
3. The problems require computation, usually multiplication and/or division.

Each of the fifth-grade pupils in the sample responded to the five problems in 40-minute sessions. The pupil responses were coded by an observer as an experimenter read the problems with the child.

The sessions were tape recorded and later transcribed. The checklist coding system developed by Days (1977), a modification of the one developed by Kilpatrick (1967), was selected for use in this study.

The data obtained from the coding were tallied to obtain the frequencies of strategy use and performance scores for each pupil. The differences in the frequencies were analyzed using X^2 statistic. Process, Range, and Performance scores were compared using t-tests of differences between means.

Results

The two groups (calculator, noncalculator) did not differ significantly on the range of processes used, time to solution, or performance.

While the data analysis are not yet complete, it appears from preliminary examination of means that the calculator group more often used the processes checks by retracing steps and compares solution with general results. Contrastingly, the noncalculator group more often selected solution on irrelevant basis and made more arithmetic errors. The calculator group used more random trial and error while the noncalculator group more often used deductive-algorithmic approaches.

These findings will be discussed in terms of effects calculator use has on thinking with implications for further research and classroom practice.

THE EFFECT OF CALCULATOR SUPPLEMENTED INSTRUCTION ON THE ARITHMETIC ACHIEVEMENT OF SECOND AND THIRD GRADERS

James M. Moser
University of Wisconsin-Madison

Perhaps the one area of agreement among reasonable debaters of the pros and cons of the use of hand-held calculators in schools is the need for further research on the effects of the use of calculators. The 1976 Conference on Needed Research and Development on Hand-Held Calculators in School Mathematics issued a call for an expanded information base regarding the use of calculators. This present study is an attempt to provide some evidence on the effects of calculator usage with primary age children.

Purpose

Specifically, the study compared the performance of children who had access to calculators for an extended period of time and whose teachers used calculators in specified instructional ways with children who did not have any access to calculators within their daily mathematics lessons. Comparison was made on the performance on experimenter-made tests of arithmetic achievement in the areas of computation and place value. The intent was to study the effect of use of calculators within the context of the regularly used classroom curriculum.

Procedure

Subjects consisted of children regularly assigned to eight classes in two schools in Madison, Wisconsin. The two schools served the same general area and were considered equivalent in terms of the population served. In each school there were two second grade classes and two third grade classes. One of each grade level was randomly designated as the experimental class and the other as the control. One school was implementing a "traditional" curriculum and the other was implementing the Developing Mathematical Processes (DMP) curriculum. DMP is based upon measurement notions and is heavily oriented towards the use of manipulative materials. The two second grade experimental classes used the Texas Instruments ABLE calculator. The ABLE is basically a four function machine with certain operations masked by means of interchangeable faces. The children used the face that masked the division key but which did show the decimal point. The two third-grade experimental classes used the Texas Instruments TI-1250 calculator which is a four-function machine with memory, change-sign and percent keys. Sufficient calculators were provided so that each child had his/her own calculator.

Since the intent was to study the effects of calculator usage in connection with an ongoing program of instruction, no specially prepared software materials were used. Each teacher was given specific suggestions on how the calculator could be used in connection with daily lessons. Suggestions included checking of answers, using the calculator as an electronic "flash card" for drill of basic facts, playing games, using repeated addition/subtraction for place value and pre-multiplication and pre-division conceptual development, and simply free play or unstructured investigation. Teachers were free to use or ignore the suggestions as they chose. Teachers kept a daily log of their activities in which type of activity, duration, number of children using calculators, and mathematical content were recorded. These data were validated by means of periodic unannounced visitations by an outside observer who independently recorded similar information. Amount of usage varied from a low of 36 percent to a high of 79 percent of the number of instructional days devoted to mathematics that the calculators were used and from a low of 16 percent to a high of 47.5 percent of the number of instructional minutes devoted to mathematics instruction that some of the children were using calculators.

The experiment began in the first week of October 1977 with the introduction of the use of calculators by the experimenter. It ended with the final post-test administered in mid-May 1978. Major tests were administered in October, February and May to all subjects. These tests covered the traditional arithmetic skills covered in second and third grades. In addition, nine shorter periodic tests were administered about every three instructional weeks to selected subjects in the experimental classes. The original intent was to administer the tests only to a randomly selected half of each class. In the two DMP classes this plan was carried out; however, in the school using the traditional program the teachers requested that the periodic tests be administered to all children. No attitudinal measures were taken.

Results

As of this writing data from the three major test administrations have not been processed except for a single overall mean score for each class for each test. In all four instances the experimental classes performed better than their control group counterparts. These results are tenuous since the intent of the major analysis is to use the pretest as a covariate in an ANCOVA. Furthermore, the single test score is a rather gross measure and the intent is to analyze data on the basis of a number of subtest scores such as simple addition facts, two-digit algorithms, place value, etc. Data from all the short, periodic tests have been analyzed, and, in general, they bear out the conclusion that the experimental classes did at least as well as control groups on measures of specific arithmetic skills. The one area where the calculator groups seem to have a statistically significant edge is place value.

Anecdotal evidence gathered by the classroom observer and provided by the calculator teachers seems to indicate that interest in use of calculators was maintained at a reasonable level after a drop from

the initial high interest that followed their introduction. No resistance was evidenced by parents or school administrators.

Conclusions

Major conclusions must await further analysis which will be completed by the end of summer 1978. However, initial considerations seem to be evident that use of the calculator did not "rot the mind" of the young users. If there are hidden insights into mathematics or much higher and positive attitudes towards the subject as the result of using calculators, this study was not designed to ascertain such results.

Reporting Session

Friday, April 20
9:00-10:00

ANALYZING COGNITIVE BEHAVIORS FOR CONJECTURING IN MATHEMATICAL PROBLEM SOLVING: AN EXPLORATORY STUDY

Linda I. Brandau
University of Illinois

and

John A. Dossey
Illinois State University

Purpose

General methods and techniques for solving problems have been discussed by mathematics educators since the time of Socrates. In many of these interchanges, problem solving has been related to one or more of the factors of creativity (Boychuck, 1974; Kilpatrick, 1969; Torrance, 1962). Other studies have focused on the heuristics used by students engaged in problem-solving activities (Boychuck, 1974; Kilpatrick, 1968; Webb, 1975) and whether or not these and related heuristics are teachable (Kantowski, 1977; Lucas, 1974).

The present study was an extension of Boychuck's work and involved the relationship between conjecturing behaviors and the creativity factors of fluency, flexibility, originality, and organization. The study also built on the work of Kilpatrick and Webb in the description of the cognitive processes employed by students involved in problem-solving situations.

The study had two general objectives: (a) to examine both the processes and the sequences of processes students used while making conjectures for five different open-ended mathematical situations and (b) to search for relationships in the clustering patterns resulting from the students' creativity scores and sequences of cognitive processes.

Procedure

In order to examine the cognitive processes employed by students in conjecturing situations, a 40-process (25 verbal, 10 nonverbal, 5 transitional) modification of the Lucas-Kilpatrick coding scheme was developed and field tested. In addition, a creativity scoring system was developed for the following factors of creativity: fluency, originality, flexibility and organization.

Thirty above-average ninth graders were individually presented with five open-ended mathematical situations and asked to think aloud while considering the problems. Their verbal comments were audio recorded. The tapes were then transcribed and the noted nonverbal behaviors were interwoven with the verbal statements. These transcripts were then

coded for the processes and scored for the creativity factors. The processes employed by each student were recorded in the sequence spoken.

These sequences were analyzed by searching for patterns of usage of: (a) the individual processes, (b) various ordered pairs of the processes, and (c) various ordered triples of the processes. Each of these three methods of analysis of the processes was further partitioned to consider the number and percentage of each process, pair, or triple for each problem, for each student, and for each student by each problem. Various cluster analyses were then carried out on the resulting data and correlational measures computed on items of interest. To aid in the analysis of the processes, they were grouped into six broad classes: Analysis/Synthesis, Interpretational, Logical, Terminal, Operational, and Procedural.

Results

Single processes.--Ten processes (3 verbal, 5 nonverbal, 2 transitional) accounted for over 70 percent of the processes coded. A cluster analysis was performed to see how the students clustered on the 20 processes most frequently used. Three clusters emerged from this analysis. The students in the first cluster used a high number of Analysis/Synthesis behaviors and a low number of Interpretation behaviors. Students in the second cluster used a high number of Interpretation processes and a low number of Terminal behaviors. The students in the third cluster used a low number of both Analysis/Synthesis behaviors and Interpretation processes.

Sequences of processes.--The 87 ordered pairs of processes which occurred 20 or more times were analyzed by focusing on the first process and its class and then on the class of the following process. The use of the "followed by" relation led to the following patterns:

> Analysis/Synthesis, Operational, and Procedural processes were most often followed by Analysis/Synthesis or Operational processes.
>
> Interpretational or Logical processes were most often followed by Terminal processes, and
>
> Terminal processes were most often followed by Operational processes.

A similar analysis was carried out focusing on the "preceded by" relation for ordered pairs. In addition the sequencing patterns for the ordered triples which were used ten times or more were studied. Several additional interesting process sequence patterns were noted.

The statistical analysis of the role of the processes within the sequences was limited to the 31 pairs and 24 triples which occurred most frequently across all problems by all students. The results indicated that the ordered pair process usage patterns were similar

($p < 0.05$) while differences existed in the ordered triple process usage patterns ($p < 0.05$).

The sequence of processes were then analyzed by problem pairs. Five of the 20 possible problem pairs showed differences in sequences when analyzed by pairs. Eighteen of the 20 problem pairs showed differences when analyzed by ordered triples of processes. An examination of individual student process usage across problems also indicated differences in individual process usage.

Creativity factors.--The transcripts were coded according to the use of four specific problem organizational heuristics. A two-way analysis of variance indicated ($p < 0.01$) that differences existed in heuristic usages across the problems. Scheffe's test was used to ferret out the differences in the organizational heuristic use across the problems.

A cluster analysis was performed using the four creativity factor (organization, fluency, flexibility, originality) scores and their sum as variables. Six clusters of students emerged. The six clusters were linearly ordered in their performance on all five creativity scores.

The cognitive process usage of students who scored one standard deviation above the mean on organization, fluency, flexibility, and organization respectively, was then compared with the process usage of the remaining students. The results of the analysis for the ordered pairs of processes did not show a significant difference for any comparison. The analysis of the ordered triples of processes showed that differences existed for the students on the variables of fluency, flexibility and originality.

Conclusions

From these, and other findings of the study, it was evident that certain types of processes were used more than others and that different mathematical content evoked different conjecturing behaviors. The study of the process sequences showed that certain individual conjecturing strategies held up across problems, while others changed. This may provide an opening to describing individual problem-solving processes, The cluster analyses tended to reinforce the individualness of the process sequences, as the clusters tended to remain far apart and contain a small number of subjects.

The creativity analysis results were less satisfying, as the creativity factors were all highly intercorrelated. Additional work needs to be done to separate the variables if they truly are factors of creativity. The analysis of the different process sequences used by the high scoring students and the other students showed that the high students tended to use more Analysis/Synthesis and Interpretation behaviors than the other students did. The use of different processes by the groups may be an indication of differential problem-solving strategies between the groups on the various factors.

Since the study was exploratory in nature, the findings must be interpreted with care. If the results hold up in replication and across different populations, it would seem that the study's findings would help prepare teachers for problem-solving instruction and for work with individual pupils.

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CONTENT RELEVANT LESSONS AND LEARNING IN HIGH SCHOOL ALGEBRA

Lyle R. Smith
Augusta College

Purpose

Bloom (1977) reviewed studies that found that the amount of time pupils spend in active learning (working on learning tasks, participating in relevant discussions, or being involved in other relevant activities) accounted for approximately 60 percent of the achievement variation of students. Shipp and Deer (1960), Shuster and Pigge (1965), Zahn (1966), and Hopkins (1966) found that mathematics teachers were more effective when they allowed their students to participate actively in developmental activities. Robitaille (1975) supported these findings.

One method teachers use to encourage pupils to be active in the learning process is to ask pupils content-relevant questions. Another method is to allow the pupils time to perform content-relevant activities. The present research was designed to investigate the joint effects of different degrees of the percentage of relevant teacher questions and the percentage of lesson time for relevant pupil activities on learning of mathematical concepts.

Procedure

Subjects.--The 350 students were enrolled in the first course in high school algebra at schools in the Austin (Texas) Independent School District. Twenty algebra teachers, all of whom had at least one year of public school experience, were randomly selected from these schools and each was asked to teach to one his or her regularly scheduled classes a 20- to 30-minute lesson on direct variation, a topic normally taught in high school algebra. The students had not previously been taught this concept in class. All 20 teachers used the same algebra textbook (Dolciani, Wooten, Beckenbach, Jurgensen and Donnelly, 1970). One month before the lessons were taught, the teachers were given a list of objectives that outlined the content on which they should focus their lessons. Specific problems in the textbook (Dolciani et al., 1970) were listed as appropriate in meeting the lesson objectives. The teachers were allowed to present their lessons in any way they chose, so long as they limited their lessons to 30 minutes.

Each lesson was tape-recorded. The students were given a 14-item test covering the topic of direct variation immediately after each lesson was presented. The teachers were not shown the test before they taught

their lessons. The split-halves reliability of the test was .83 after the Spearman-Brown correction formula was applied.

In an attempt to equalize classes in terms of mathematical ability, the California Achievement Test (CAT) percentile rank mathematics scores of the students were used to adjust raw posttest scores through an analysis of covariance. The covariance-adjusted scores were used as the criterion of teacher effectiveness.

Two variables pertaining to pupil classroom activities were investigated by analysis of the tape-recorded lessons. One variable, On Task Questions, was the percentage of teacher questions that were directly related to the objectives each teacher was given. Three levels of this variable were defined. One level, referred to as HQ, required 100 percent of the teacher questions to be on task during the lesson. The second level, MQ, required that between 67 and 100 percent of the teacher questions be on task. The third level, LQ, required that 0 to 67 percent of the teacher questions be on task.

The second variable, On Task Activities, was the percentage of lesson time each teacher allowed for students to work problems individually or in small groups, to ask questions concerning problems involving direct variation, or to make up problems involving direct variation and present them to the class. Three levels of this variable were defined. One level, referred to as HA, required that the teacher allow at least 20 percent of the lesson time to be devoted to on task activities. The second level, MA, required that between 10 and 20 percent of the lesson time be devoted to on task activities. The third level, LA, required that 0 to 10 percent of the lesson time be devoted to on task activities.

The levels for both variables were defined after the tape-recorded lessons were analyzed, since it was not feasible to determine beforehand the degrees of variability that would occur.

Results

A 3(HQ vs. MQ vs. LQ) x 3(HA vs. MA vs. LA) least-squares analysis of variance was performed on the covariance-adjusted student achievement scores. The least-squares method was used since the sizes of the experimental groups varied so greatly. The mean achievement scores and the frequencies for the cells, as well as the number of classes that represent each cell, are presented in Table 1. Table 2 indicates that the On Task Questions main effect was significant, $F(2, 341) = 38.35$, $p < .001$, as was the On Task Activities main effect, $F(2, 341) = 8.24$, $p < .001$. The interaction between On Task Questions and On Task Activities was also significant, $F(4, 341) = 3.55$, $p < .01$. Simple main effects comparisons and the Newman-Keuls method indicated that the following results were significant beyond the .01 level: HQ > LQ, MQ > LQ, HA > MA. At the .05 level, HQ > MQ, HA > LA. These relationships are illustrated in Figure 1.

Table 1

Group Frequencies and Covariance-Adjusted Means

	HQ	MQ	LQ
HA	2 Teachers n = 34 $\bar{X}$ = 11.838	2 Teachers n = 43 $\bar{X}$ = 11.128	2 Teachers n = 35 $\bar{X}$ = 10.017
MA	3 Teachers n = 52 $\bar{X}$ = 11.172	3 Teachers n = 41 $\bar{X}$ = 10.641	2 Teachers n = 38 $\bar{X}$ = 7.291
LA	1 Teacher n = 24 $\bar{X}$ = 11.687	3 Teachers n = 47 $\bar{X}$ = 9.960	2 Teachers n = 36 $\bar{X}$ = 8.947

Table 2

Results of the Analysis

Factor	df	SS	MS	F
On Task Questions (Q)	2	448.7	224.35	38.35**
On Task Activities (A)	2	96.5	48.25	8.24**
Q x A	4	83.2	20.80	3.55*
Error	341	1995.2	5.85	

* $p < .01$

** $p < .001$

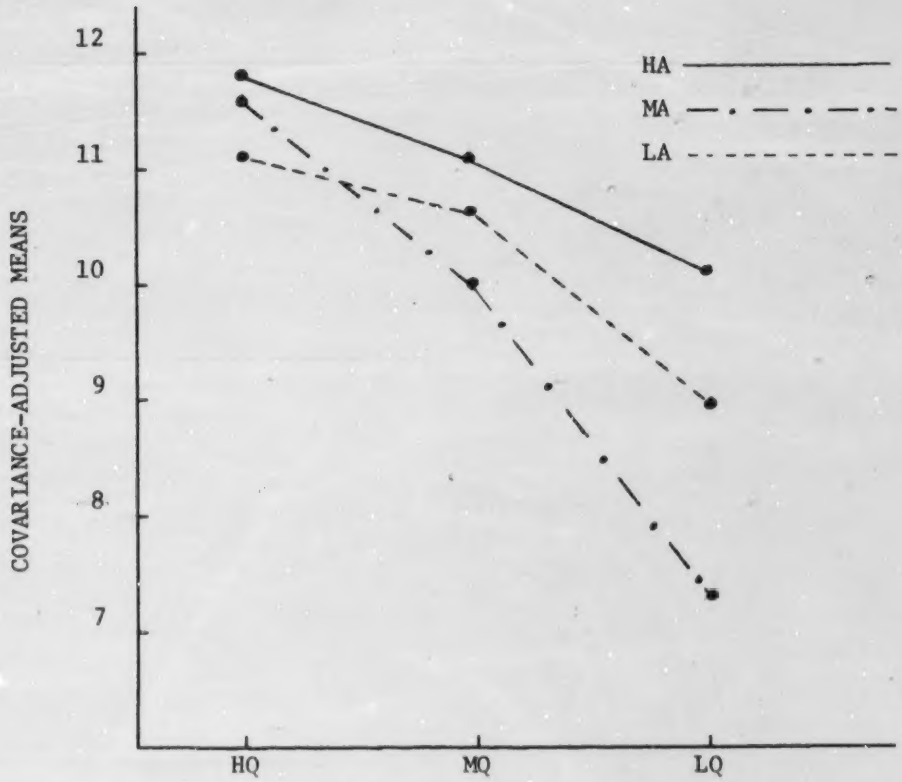

Fig. 1. Covariance-adjusted means for treatment groups.

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THE EFFECTS OF SPATIAL INSTRUCTION ON PROBLEM-SOLVING PERFORMANCE

Barbara Moses
Indiana State University, Evansville

Purpose

Current research studies concerning brain hemispheric activity have indicated to educators that an insufficient amount of time is being devoted to the development of the right brain hemisphere which is responsible for spatial thinking.

This investigation attempted to analyze the effects of a series of lessons which were designed to encourage a spatial mode of thinking. The following general questions were the foci of the investigation:

1. What effect, if any, will spatial instruction have on problem-solving performance?

2. What effect, if any, will spatial instruction have on the solution processes used in problem solving?

3. Is there any significant aptitude-treatment interaction, that is, will the instruction affect students of high spatial ability differently than students of low spatial ability and will it affect students of high reasoning ability differently than students of low reasoning ability?

4. Is there any significant sex-treatment interaction, that is, will the instruction affect males differently than females?

Procedure

The study was carried out over a 15-week period during the second semester of the 1977-78 academic year. The sample population consisted of four intact fifth-grade classrooms, for a total of 132 students. All four classes were given the pretest and posttest batteries during the regular mathematics period; two of the four classes were randomly selected to serve as experimental classes and received the spatial instruction, presented by the investigator during the regular mathematics period.

The pretest and posttest batteries consisted of the same set of seven tests. Four of the tests were spatial ability tests that had been

selected based on a previous research study by the investigator. The four spatial tests chosen were: Punched Holes Test, Card Rotations Test, Form Board Test, and Mental Rotations Test. Two of the tests in the battery were logical reasoning tests which were selected based on pilot-testing. These two tests were Nonsense Syllogisms Test and Inference Test. The seventh test was a Problem-Solving Inventory compiled by the investigator which consisted of ten non-routine problems.

Four scores were computed for each student from both the pretest and posttest batteries:

a. Spatial ability score--This measure of spatial ability was computed by summing the z-scores on the four spatial tests.

b. Reasoning ability score--This measure of logical reasoning ability was computed by summing the z-scores on the two logical reasoning tests.

c. Problem-solving performance score--This measure of success at problem solving was computed by counting the number of correct responses to the 10 problems on the Problem-Solving Inventory.

d. Degree of visuality score--This measure of the amount of spatial skills (use of pictures, graphs, tables, constructions, etc.) was computed by assigning a score of 0 to 2 for each item on the Problem-Solving Inventory and then summing across the 10 items.

The pretest battery was administered during the first week of the study. The instructional period then lasted for 13 weeks and was followed by the posttest battery which was administered during the final week of the study. The instruction included five weeks of lessons with three-dimensional objects (using D-stix, pipe cleaners, clay, and construction paper), five weeks of lessons with two-dimensional objects (using tangrams, geoboards, georules, and construction paper), followed by three weeks demonstrating the relationship between two-dimensional and three-dimensional objects.

Results

1. The instruction significantly affected problem-solving performance; the effect was more significant on problems which were more spatial in nature than on problems which were more analytic in nature. These results were based on analyses of covariance, using pretest measures as covariates, in order to factor out any initial differences between control and experimental classes.

   These findings appear to confirm the notion that spatial instruction is a lacking element in the teaching of mathematical problem solving and can be taught effectively in the classroom.

2. The instruction did not significantly affect the solution processes used in problem solving, as manifested on test papers. Further investigation by individual interviews, however, showed that students were using spatial techniques to a greater extent, but they did not write these processes on their papers. (This may be due to prior training.)

   This result was based on analysis of covariance, as previously described.

3. There were no significant differences in instructional effects in problem-solving performance between students of high and low spatial ability. However, there were significant differences in instructional effects in problem-solving performance between students of high and low logical reasoning ability; students of low reasoning ability profited more from the instruction. These results were based on analysis of variance, using students from the experimental classes only.

   These findings seem to strengthen the conclusion stated in finding #1. The student who has found no success in an analytic mode of instruction has no other tools upon which to rely. Supplying such a student with spatial skills increases his probability of success greatly.

4. There were no significant differences in instructional effects in problem-solving performance between males and females, based on analysis of variance using experimental classes only.

   This finding contradicts previous research which indicated that females are lower in spatial ability and problem-solving performance due to their lack of experience. Certain researchers had hypothesized that, given the proper training, females could be brought up to the level of males. This investigation did not confirm this hypothesis.

DEVELOPMENT AND VALIDATION OF A TESTING INSTRUMENT FOR PROBLEM SOLVING STRATEGIES OF CHILDREN IN GRADES FIVE THROUGH EIGHT

Theresa Oehmke
University of Iowa

Purpose

The four-stage problem-solving process proposed by Polya has often been used by researchers in their studies of mathematical verbal problems. A paper-pencil test was designed to provide individual student profiles which illustrate the performance at each stage of the process. Thus, the primary purpose of this study is the validation of this test and its subtests of the four stages of the problem-solving process.

Procedure

Based on an extensive search of the related literature, it was concluded that this is the first attempt to test these four stages in a multiple choice format. Thus, much effort was required to design items to measure the appropriate knowledge and skills at each stage of the problem-solving process. After a good deal of "brainstorming" with members of the Iowa Problem Solving Project* (IPSP), a search of the problem-solving literature, and a detailed analysis of each stage, the following testing model was chosen.

I. Get to know the problem
   - A. Determine insufficient information
   - B. Determine extraneous information
   - C. Write a question for the problem setting

II. Choose what to do from a list of strategies

III. Do it
   - A. Choose the necessary computation
   - B. Estimate from a diagram
   - C. Compute from a diagram
   - D. Use a table
   - E. Compute from an equation

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*The IPSP is a federally-funded research and development project directed by George Immerzeel. The staff is composed of five university and elementary school mathematics educators.

IV. Look back

A. Relate to problems that can be solved in the same way as a given one
B. Vary conditions in a given problem
C. Check a solution with the conditions of the problem

An example of an item from IA of the list is:

> Eva knew that Babe Ruth had hit 714 home runs in his major league baseball career. She wanted to know how many home runs he averaged per year. Which choice below would she also need to know about Babe Ruth?
>
> 1) He hit 60 home runs in 1927.
> 2) He played for 20 years.
> 3) He died in 1948.
> 4) He was a pitcher for three years.

After several tryouts and revisions the individual item statistics were found to be satisfactory. The measurement characteristics of the test (and in this case, its four subtests which measure each stage of the problem-solving process) were then determined. The first aspect, reliability, was computed for the entire test and for each subtest using a representative sample of Iowa fifth, sixth, seventh and eighth graders.

The second aspect, test validity, is probably the most important characteristic of a good test and the most difficult to ascertain. The simplest type of validity, face validity, was established by the IPSP staff. That is, all the staff agreed that the items are measuring the proficiency of the student at each stage of the problem-solving process.

However, for a more critical judgment of the test's validity, the following question was asked. To what extent would the results of the paper-pencil test correlate with the results of a scoring scheme used to analyze interviews? Such a scheme was designed for measuring the problem-solving behavior of students in a one-to-one "think aloud" mode. Thus the relationship of the two measures, paper-pencil and interview, will be one estimate of the concurrent validity of the problem-solving process test. Another estimate of concurrent validity will be derived from the test's relationship to several standardized achievement measures. Concurrent validity is being investigated via three different studies.

Study One: Thirty-two children in grades five through eight (8 per class) completed the paper-pencil test and also completed from 6 to 15 verbal problems in a "think aloud" interview. In order to neutralize the practice effect, half the subjects took the paper-pencil test before the interview and the other half were treated in the reverse order. Also, in an attempt to facilitate the student's verbalization of his thought processes, the problems were selected

from a set of 65 problems which were grouped into seven levels. Each student's problem sequence was determined by his ability to react and verbalize to the previous problem. The results of the interviews are being analyzed from tapes and notes. Their relationship with the results of the paper-pencil test will then be determined.

Study Two: Fifty-five children in grade five from another school completed the paper-pencil test. Then the same (in contrast to Study One) set of five verbal problems was administered to all the children in a "think aloud" interview. The pencil-paper data have been analyzed and the interview tapes are now being processed.

Study Three: A representative sample of 2107 students in grades five through eight of the Iowa school population completed one of six parallel forms of the pencil-paper test. The strength of the relationship between this test and four scales on the Iowa Test of Basic Skills (reading comprehension, graphs, arithmetic concepts, and arithmetic problems) is being analyzed.

Results

Results from all three studies will be reported at the NCTM meeting.

THE HAND-HELD CALCULATOR IN THE ELEMENTARY SCHOOL, AN EXPLORATORY STUDY OF TWO ISSUES: DEPENDENCY AND THE EFFECT ON THE PROBLEM-SOLVING PROCESSES

Marguerite Mason
University of Iowa

Purpose

Within the last few years, hand-held calculators have become readily available; in fact, almost commonplace in our society. Elementary school teachers must face the fact that many of their students are familiar with calculators and will be using them outside their classrooms. These teachers have at least three choices: refuse to allow calculators in their classrooms, allow their students to use calculators in their classrooms in a limited way such as to check answers only, or integrate calculators into their curriculum fully wherever appropriate.

Before they can decide which route to take, the question of what effects these calculators have on learning mathematics in elementary schools needs an answer. The NSF Project "Impact of Calculators in Elementary School Mathematics" has been exploring these effects. Mark Twain Elementary School in Iowa City, Iowa is one of five sites participating in the project. One classroom at each grade level 2-6 used TI-1000 calculators as part of their mathematics instruction while another class at each of those grade levels served as a control without calculators. The experimental and control classes were tested in October, February (before the roles were reversed), and May to assess the effects on basic facts, computation, learning of concepts, applications, and attitudes. So far, no significant difference between the two groups has been found.

The research to be reported on was done in conjunction with this NSF Project. These are several questions that cannot be readily answered by the use of a conventional standardized test instrument. Yet they are questions that we should attempt to answer before we can decide to what extent calculators should be used in the mathematics classroom such as:

1. Do elementary school students who are allowed to use hand-held calculators in their mathematics classes over a period of several months become dependent upon them?

2. Does the availability of a hand-held calculator change a student's algorithmic approach to a problem-solving situation over the course of several months' experience?

Procedure

Two pairs of students, one of boys and the other of girls, matched on the basis of pretests on basic facts and the math concepts and math applications sections of the Standford Achievement Test, were chosen at each grade level 2-6. One student from each pair was from the experimental class and the other from the control class. Each of the 20 students was given a set of computer-generated computation exercises and five word problems to solve in an interview situation approximately once a month throughout the 1977-78 school year. During the entire series of interviews, all subjects had a TI-1000 calculator and a supply of paper and a pencil available. On the computational exercises careful note was made not only of the subject's answers, but also of the mode (calculator, paper and pencil, or in his head) each chose for each problem. An efficiency score for each mode for each problem type was determined. One unexpected aspect emerged: the mode each individual student chose for a particular exercise during the first interview before any of the subjects had been introduced to the calculators predominated as the mode that student used for the same type of problem throughout the remainder of the six interviews, no matter which group the student was in.

To explore how the availability of a calculator during their mathematics instruction affects a student's algorithmic approach to a problem-solving situation, the students were asked to set up and solve five word problems appropriate to their grade level using the "think aloud" technique. Tape recordings of these interviews were analyzed using a coding form similar to that developed by Jeremy Kilpatrick (1967) but modified for calculator availability. The responses of the students in the experimental (calculator-involved) groups have been compared and contrasted over the 15-week period as they became more experienced with the calculator. The effect of maturing and regular mathematics instruction has been assessed by comparing the approaches of the calculator and the non-calculator groups during this period. The effect of the calculator usage is also observable during the eight-week period at the end of the school year in which the non-calculator group received the modified calculator treatment.

At the conclusion of the project in May 1978, similar interviews with the entire third and fifth grades were conducted. Half of the experimental group and half the control group were randomly assigned to have calculators and paper and pencil available during the interview while the remainder of the students had only paper and pencil available.

Results

The primary statistical analysis was a three-factor fixed effects analysis of variance. The three factors include: Treatment (two levels), Grade (two levels), and Calculator Availability during Interview (two levels). There was no evidence that the students became calculator dependent. Follow-up tests showed that it was advantageous to have the calculator available for certain types of

exercises such as division for the third graders. The analysis of the coding forms for the word problem solving is still under way.

The large group interview indicates that there are some types of exercises with which it might indeed be beneficial to use calculators in instruction and that calculators do not "rot the mind" as some critics have feared.

The information obtained from the seven interviews with the 20 students over the course of the year is not meant to be a generalizable study, but it has given some clues as to how problem-solving approaches evolve and how they are affected by calculator usage. It also points the way to further needed research.

Reference

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AN INVESTIGATION OF TWO PRESENTATION MODES ON DYNAMIC PROBLEMS IN ALGEBRA IN RELATION TO APTITUDES AND SEX

Patricia A. Semmes
University of Texas at San Antonio

and

Ralph W. Cain
University of Texas at Austin

Purpose

Current research in mathematics education includes the investigation of presentation modes of verbal problems, the search for aptitude-treatment interactions, and the examination of sex differences in the learning of mathematics. This study investigated two modes of presenting dynamic multivariable verbal problems in relation to the aptitudes of spatial visualization and general reasoning and in relation to sex. One presentation mode utilized motion pictures via film to simulate the action in the problems. The other mode utilized still pictures via slides in which the action was implied by the sequence of still pictures but could not actually be viewed.

Procedure

The Paper Folding Test and the Necessary Arithmetic Operations Test obtained from the Kit of Reference Tests for Cognitive Factors were used to measure spatial visualization and general reasoning, respectively.

A 21½-minute 16mm film with sound and a parallel slide-audio presentation containing 178 slides were developed by the principal investigator. Uniform motion problems commonly found in the ninth grade algebra curriculum were selected for the treatments, and some instruction, based on Polya's heuristics for problem solving, was provided. The treatments were aimed at simulating the action within the parts of the problems in order to help students to determine the implied conditions, especially those relating to distance. Algebraic solutions were not included.

The criterion instrument was constructed by the principal investigator and contained four learning and four transfer problems. Depending on the problem, students were to choose the open sentence which described the implicit relationship between the distances traveled by the objects, to select the diagram which showed the relationship between the distances traveled by the objects after a stated amount of time, to pick out the open sentence which described the explicit relationship

between the times that the objects traveled and/or to select the best estimate of the answer. Solutions to the problems were not required.

The tests and treatments were administered to all of the ninth grade algebra I classes (8 classes, 208 students) at a public school in a suburb of Dallas, Texas, in the Fall 1976. Students within each class were randomly assigned to treatment and viewed the appropriate presentation as part of the regular work prior to the unit on motion problems. The experiment was conducted in two consecutive class periods with the aptitude tests being administered during the first class period and the treatments and criterion test being administered during the second class period.

The hypotheses were tested using multiple linear regression, and Johnson-Neyman techniques were used to compute the regions of significance for disordinal interactions. Hypothesis 1 dealt with a gross comparison between the two presentations:

1. The mean achievement test score for the film group will not be significantly different from the mean achievement test score for the slide group.

Hypotheses 2 and 3 related to the ATI effects:

2. When scores on the achievement test (following presentations on dynamic problems via film or slide) are regressed on scores on the measure for spatial visualization, the regression lines across treatments will be parallel.

In cases where the results of the hypothesis testing warranted further investigation, the appropriate hypotheses were re-tested independently for subpop ations of males and females. Hypotheses 1 and 2 were retested ir s manner.

Results

The results of testing hypothesis 1 indicated that the film was more effective ($p < .072$) than the slide presentation. Further testing revealed that the film was superior to the slides in the male subgroup but not in the female subgroup.

Tests of hypotheses 2 and 3 yielded no interaction using general reasoning as a predictor, but a strong disordinal interaction ($p < .004$) was found using spatial visualization as a predictor. Forty-five percent of the observed data points were within the region of significance. Further analysis showed a strong disordinal interaction within the female subgroup. In this subgroup 30 percent of the observed data points were within the region of significance. No interaction was found in the male subgroup.

Conclusion

The results suggest that motion picture simulations of dynamic multivariable verbal problems are more effective for males than simulations utilizing still pictures whereas for females the mode of presentation interacts with spatial visualization ability. They further suggest that within the female population motion picture simulations are more effective for good visualizers, while still pictures are more effective for poor spatial visualizers.

THE EFFECTS OF SITUATIONAL PROBLEM-SOLVING AND ELECTRONIC CALCULATING INSTRUMENTS IN A COLLEGE LEVEL INTRODUCTORY STATISTICS COURSE

Sharon Whitton Ayers
Mercer University in Atlanta

Purpose

This experimental study focused on determining the effects of situational problem-solving heuristics and electronic calculating instruments on selected cognitive and affective variables in a college level introductory statistics course. In particular, this study was designed to detect differences and associations with respect to the following areas: (1) mastery of statistical concepts, (2) attitude toward mathematics, and (3) general view of statistics. The primary objective of the study was to evaluate a teaching method developed to increase student awarenesses of statistical concepts and skills through student involvement in realistic problem situations. This teaching method stressed problem formulation and mathematical modeling processes. A second objective was to determine the effects of classroom access to the following two electronic calculating aids: (1) the Compucorp Micro-Statistician Calculator, Model 342 and (2) the Automated Statistics Laboratory (ASL) computer package.

Procedure

The 172 subjects who participated in the study were enrolled in an introductory statistics course at Georgia State University designed for non-mathematics and non-science majors. The only prerequisite to the course was one year of high school algebra.

Based on the outcomes of a pilot study involving 48 students, an experimental study spanning eight weeks was conducted. The four classes comprising the experimental study corresponded to two instructional modes and two computational methods. The two levels of instructional mode included emphasis on situational problem solving (S) versus no such emphasis (NS); the two levels of computational method included classroom access to electronic calculating instruments (E) versus no such access (NE). Subjects were randomly assigned to one of four combination treatment conditions: S-E, S-NE, NS-E, NS-NE.

The dependent variable, statistics content achievement, was measured by an investigator-developed instrument Statistics Content Assessment (SCA). Data analysis involved use of a two-way ANOVA to determine main and interaction effects of instructional mode and

computational method on statistics content achievement. Additionally, subjects were administered posttests on the McCallon and Brown Attitude Toward Mathematics (MBAM) scale, and the investigator-developed View of Statistics Inventory (VSI). Differences and associations with respect to the above three criterion measures were analyzed by the Mann-Whitney U and the Kendall Tau tests, respectively. Ten hypotheses were formulated and tested which interrelate the criterion measures with instructional mode and computational method.

A post study was conducted the following quarter to determine the replicability of procedures and results.

Results

The main effect of computational method on content achievement was highly significant ($p < .001$) with the mean SCA score of the E group exceeding that of the NE group. There was no discernible difference in the mean SCA score between the S and NS groups nor was there a significant interaction effect between instructional mode and computational method. Significant positive associations ($p < .001$) were found between (1) the SCA and the MBAM, (2) the SCA and the VSI, and (3) the MBAM and the VSI. There were highly significant main effects ($p < .001$) of instructional mode on both attitude toward mathematics and view of statistics with the S group exhibiting higher MBAM and VSI scores than the NS group. There was no discernible difference in either attitude toward mathematics or view of statistics between the E and NE groups.

Results of the post study ($N = 24$) involving the combination treatment S-E resulted in a mean SCA score of 118.96 as compared with the mean SCA score of 118.28 computed for the S-E group in the experimental study.

Conclusions

This study supplies evidence to support classroom use of electronic calculating instruments in an introductory statistics course. Moreover, instructional emphasis on situational problem-solving heuristics resulted in significantly higher attitudes and view of statistics while not interfering with acquisition of standard course content. The feasibility of replicating these experimental results is substantially supported by the near equivalent SCA mean achievement scores for the corresponding experimental and post study groups.

Reporting Session

Friday, April 20
2:30-3:30

THE CALCULATOR'S EFFECT ON CHILDREN'S SOLUTION PROCESSES

Glendon W. Blume
University of Wisconsin-Madison

Purpose

The adoption of the hand-held calculator (HHC) for use in the classroom represents a package deal. There are areas of the curriculum in which the HHC may be used to great advantage. There are other areas for which the HHC may have few, if any, implications. However, the total picture is not a simple dichotomy. There is a third domain in which use of the HHC may lead to undesirable consequences or unnecessary procedures. The purpose of this abstract is to describe research aimed at identifying and investigating this third area. It is based on pilot work during the spring of 1978 and continuation of that research during the fall of 1978.

Perhaps an illustration will serve to provide the reader with a flavor for the purpose and scope of this investigation.

> Jack bought 16 used bicycles at $5.00 each. He cleaned them up and sold them for $11.00 each. What was his profit?

Two distinct strategies for solving this problem can be readily identified, and both involve distributivity of multiplication over subtraction:

(a) $16\ (11-5) = X$ (b) $(16 \times 11) - (16 \times 5) = X$

There are questions which can be raised concerning differences in children's performance on this problem if using the HHC or if using only paper and pencil (P&P) computation. Children who must use P&P might benefit from thinking about various solution methods before proceeding with the computation. The time spent in thought may be compensated for by discovery of an efficient and direct solution method. Therefore, the first major question is, If given the exemplar above, will children approach it differently if they have an HHC or P&P available? If children do give more thought to various solution methods when they are working with P&P only, then the next question is to determine if this additional thought is educationally significant. If so, one must then focus on identifying areas of the mathematics curriculum in which such additional thought is desirable and likely to occur.

Another question concerns a change in performance accompanying a change from calculator use to P&P use. The following is another exemplar used in this study.

> The star basketball player scored a total of 297 points during the first nine games. Jill averaged 14 points fewer per game than the star. How many points did Jill score during the first nine games?

Solution methods:

(c) ((297 ÷ 9) - 14) x 9 = X (d) 297 - (14 x 9) = X

Results

Preliminary findings indicate that children in the sample (21 seventh-grade pre-algebra students) tended to use different solution methods for both of the above exemplars when working with the HHC than were used when restricted to P&P. Each subject was given two equivalent five-item tests, one with the HHC available and one with P&P available; order was controlled by randomly assigning half of the Ss to use P&P first. On one of the two five-item tests, all HHC solutions for both of the above exemplars involved the longer solution, (b) or (c), while the P&P solutions were split between the longer and more direct methods. In addition, of seven Ss who changed solution methods across the use of different aids (HHC or P&P), six used the more direct approach when given P&P and the longer method when using the HHC.

The study described above was exploratory and the results are not conclusive. However, enough difference in solution methods was exhibited to warrant further investigation. A follow-up investigation is planned (for Fall 1978) in which a more sensitive instrument will be used; test items will be expanded to include a wider range of problems for which different solution methods exist. Subjects in middle school, high school, and college will be tested in an attempt to obtain a broader cross-section of subjects, and individual interviews will provide additional insight into solution methods used.

The HHC is a complex, totally symbolic, manipulative device, and the implications of its use will prove to be equally complex. This investigation is intended to uncover some of these implications, in particular those concerned with the HHC's effect on solution methods used by students. McClintock (1978) addressed this broader issue in recommending:

> The need for examining the process dimension of problem solving, particularly in a clinical-exploratory nature is, therefore, a vital area of calculator research that remains virtually untouched (p. 5).

It is anticipated that the present study can begin to contribute to identification of a class of heuristics; e.g., use of guess-and-test, application of a structure property such as distributivity (in search for a more direct solution, etc., which are calculator enhanced or calculator inhibited.

Reference

McClintock, C. E. The hand calculator as an Algebra I problem-solving tool. Paper presented at the National Council of Teachers of Mathematics Annual Meeting, San Diego, California, April 12-15, 1978.

Reporting Session

Friday, April 20
4:00-5:00

THE CHILD'S CONCEPTUALIZATION OF SUBTRACTION

Charles E. Mitchell
University of Wisconsin

Purpose

This manuscript describes ongoing research being conducted by the author. The general purpose of this research is to further our knowledge of a child's conceptualization and understanding of the operation of subtraction. A part of this investigation will focus on the child's awareness and understanding of the non-commutativity of this operation.

The fact that subtraction is a problem area for many elementary school children has been verified by many researchers. The reader is referred to studies by Roberts (1968), Cox (1974, 1975), Pincus et al. (1975), and Carpenter et al. (1975). A report from the National Assessment of Educational Progress (1975) and another by Carpenter et al. stated that "for 9-year-olds, the most common mistake was a (subtraction) reversal error," i.e., reversing parts of the minuend and subtrahend. Although this error occurs within an algorithm for compound subtraction, it may be possible to trace its cause to some early conceptual difficulties.

Procedure

In order to obtain some preliminary indications of children's conceptualization of subtraction, 50 children were randomly selected from the population of a small, private elementary school and interviewed by the author. Ten children were selected from each of grades one through five. Table 1 is a complete list of the stimuli used to elicit responses from the children, with a period of questioning following the items to better determine the child's thinking. Children were instructed to read each item aloud, and to write down the answer, if they thought they knew it. To control for order effects, parts one and two were alternated throughout the administration of the instrument.

Table 1

The Test Items

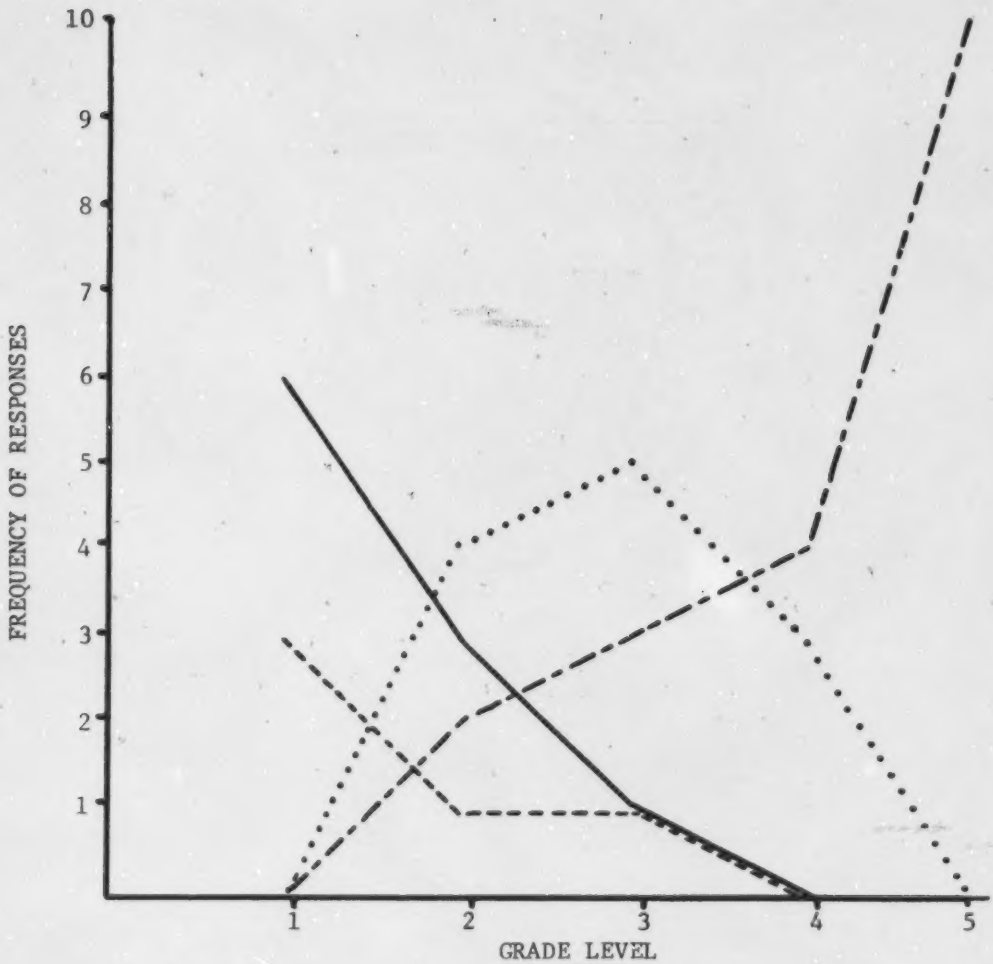
Part 1:	6 - 9 = __	_____
	8 - 5 = __	_____
Part 2:	7 - 3 = __	_____
	3 - 5 = __	_____

Results

The responses of the children to the two items with negative solutions are summarized in Table 2. For the other two items, only one child in first grade could not answer both correctly. Generally, children in first grade responded 'zero' to both items with negative solutions, with nine of the ten children responding 'zero' to at least one of them. Three of the children seemed to use different strategies to answer these two items by responding '+2' or '+3' to one of the items and 'zero' to the other. These children, and others like them in the other grades, are placed in a separate category in the table.

Table 2

Responses to 3-5 = __ and 6 - 9 = __



——— Responded 0 to both number sentences
– – – – Used two different strategies to solve the number sentences
—·— Stated that the problems cannot be solved
• • • • • Responded +2 and +3 to the number sentences

Note: Three children in grade four gave the correct response.

If we shift our attention to grade three, we see an entirely different picture. These children were more likely to respond '+2' or '+3' to the items in question. The next most popular response, which grew in popularity across grades, was that the items with negative solutions could not be solved. The 'zero' response had almost vanished by third grade.

By fourth grade, the most popular response was that the two items could not be solved, with the '+2' and '+3' response still popular. By grade five, all children stated that the items could not be worked. Some of these children assumed that the author had made a mistake and reversed the order of the numbers. This was one reason that follow-up questioning proved important.

For the most part, the teachers had given the children no instructions as to how these items should be treated. However, during regular instruction, when children translated word problems into symbolic form, they were corrected if the order of the numbers in the subtraction and division equations written by the children was not correct. An example of this would be the following. Jack lost 3 marbles. If he had 5 to start with, how many are left? If the child wrote '3 - 5 = 2,' then the teacher would correct the order.

Extension

Future research will consist of two steps. The first step will be to replicate the previous study in another school. However, the questioning will go into more depth and attempt to answer various hypotheses about the reasons that children are responding as they are. A sample of these hypotheses are listed for the reader.

H_1 - Are children who respond 'zero' to items with a negative solution using a concrete 'take-away' model when they subtract?

H_2 - Are children who respond '+2' or '+3' to the items in Table 2 manipulating symbols rather than using a concrete model?

H_3 - What percent of the children who make the '+2' or '+3' response actually believe that subtraction is a commutative operation? Some children in the previous study justified a response of '+2' to '3 - 5 = __' by noting that 2 + 3 = 5. This is the additive check of the solution. Others simply stated that order did not matter; i.e., the commutative response.

The first step of this research will be followed by a treatment in order to determine if the children's response patterns can be changed to a more desirable pattern. It is important for children to realize that order does matter in subtraction.

The treatment will consist of a study of some basic models for the negative integers such as owing, temperature, and being in the hole. These models will be introduced to children in grades one through three. Some preliminary piloting indicates that most of these children are quite capable of working with these basic concepts.

Working with these models may also have a desirable effect on children's performance in constructing symbolic equations from word problems. It may be possible to make children more aware of the effects of order without actually stressing work with symbolism.

In summary, many interesting results may follow as a result of this work. All research should be completed during the fall of 1978 and available for dissemination at the annual meeting in Boston.

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A PROCEDURE FOR IDENTIFYING THE COGNITIVE PROCESSES USED DURING PROBLEM SOLVING*

Frank K. Lester
Indiana University

Purpose

The cognitive processes used by children during problem solving have received considerable attention in mathematics education research in recent years. The identification of the processes used by both successful and unsuccessful problem solvers has been hampered by the difficulties involved in making accurate observations of mental processes which may or may not have physical manifestations during problem solving. In addition, problem solvers themselves often are unaware of many of the thought processes they use while attempting to solve problems. This state of affairs points to the following questions:

1. Can a valid and reliable procedure be devised to observe the cognitive processes used during problem solving?

2. What are the effects of externally provided cues on the processes used during problem solving?

3. To what extent do problem solvers continue to rely on trial-and-error behavior as the amount of available information increases?

4. Does trial-and-error behavior decrease as the problem solver's age increases?

5. What is the relationship between ability to formulate, test, discard, and reformulate hypotheses and grade level (age)?

Thus, this study focused on the identification of the processes used by problem solvers without prior instruction and the ways in which systematically provided information affects their problem-solving behavior.

Since problem solving is viewed as such an important part of mathematics, it is imperative that more effective instructional techniques be developed. Clearly, problem-solving instruction must be based on knowledge of the capabilities of the students involved. Consequently, there is a need to determine which abilities children possess naturally

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*This study was supported by a grant from the Spencer Foundation.

and which ones they can be taught to use effectively. Recent research efforts have focused on "ecological experiments" which attempt to study the relations between the characteristics of learners and their environments. (See Bronfenbrenner, 1976; Hatfield, 1976; and Kilpatrick, 1978; for discussions of this shift in methodology.)

One aspect of this new trend in problem-solving research has been the attempt to conduct qualitative analyses of problem-solving behavior in addition to, and in several cases instead of, focusing only on quantitative analyses. This emphasis recognizes the likelihood of gaining valuable understanding of subjects' problem-solving behaviors by witnessing their cognitive processes as they may be developing.

Unfortunately protocol analysis based on "thinking aloud" or retrospective approaches has serious weaknesses. This approach can be attacked on the grounds that forcing the subject to think aloud during problem solving may have a deleterious effect on performance. At the same time there is a tendency for subjects to talk only about moves which they believe to be safe or correct. Retrospective analysis is often criticized because it is unreliable inasmuch as it is unlikely that subjects can reconstruct accurate accounts of their behaviors including all of the cognitive processes employed.

An ideal situation would be to devise problem-solving tasks which force the subject to manifest mental processes through the physical manipulation of objects thereby enabling an observer to record the processes being used. Thus, the need for the subject to verbally relate her/his thoughts or accurately remember what was done would be removed. One such task has been developed by this investigator. The value of the task lies in the possibility of a very detailed qualitative analysis of the subject's cognitive processes. The nature of the task allows subjects to be comparatively free in their procedures for attacking the problem, but at the same time the physical moves made reflect nearly every step in their reasoning: forming hypotheses, testing and discarding them, attention to relevant cues, and so on. The whole course of the task: comprehension of the problem, testing conjectures and responding to additional information, and ultimately finding a solution—provides a number of indicators of problem-solving ability or the lack of it.

Procedure

Materials used in the experimental task consist of 22 wooden blocks varying in color, shape, thickness and size. There are five colors, six shapes, two thicknesses, and two sizes. On the underside of each block which is not seen by the subject, is written one of four letters: A, B, C or D. Regardless of color and shape, A is written on all large, fat blocks; B on all large, thin blocks; C on all small, fat blocks; and D on all small, thin blocks. At the beginning of the experiment all blocks are scattered randomly on a table top. S is told that there are four different letters written on the underside of each block and that all the blocks having the same letter written on them are alike in some way. The task is to attempt to put the

blocks into four piles so that all the blocks in a given pile have the same letter written on them. E then turns up one of the blocks, shows the letter on it to S, and, putting it in one corner of the table, suggests that S start by picking out and placing in the same corner all blocks which might belong in the same pile as the block which was turned up. After S does this, selecting, for example, all blocks of the same color as the sample block, E turns up one of the wrongly selected blocks, showing that this is a block of a different kind. S is encouraged to keep trying. After each new attempt, S is asked to explain the new grouping and one of the wrongly placed blocks is turned up. The procedure continues until S discovers the principle of classification or all the blocks have been turned up by E in the process of correction. Throughout the experiment a detailed record is made of each move S makes, each correction E provides, and all verbal comments made by E and S. The task will be presented to Ss on a one-to-one basis.

Although the task is primarily a tool for qualitative analysis, for purposes of age group (grade) comparisons a quantitative measure of performance will be determined. Time needed for solution and the number of corrections (i.e., the number of blocks turned up by E) will be the two measures used for this analysis.

Tentatively, four groups of subjects will be included in the study. Twenty-five Ss will be selected from each of grades 4, 6, 8 and 10, representing stages of cognitive development from concrete operational to formal operational thought. It is expected that this six-year range in age will provide ample opportunity to identify developmental changes in problem-solving behavior. Task will be administered and data analyzed during the fall semester, 1978.

Anticipated Results

Although specific results cannot be determined with any certainty, the following conjectures about the results are made on the basis of preliminary study by this investigator and knowledge of children's problem-solving capabilities:

1. There will be dramatic developmental differences with respect to:

   a. use of trial-and-error behavior (it will decrease with increasing age);

   b. ability to coordinate multiple attributes (younger Ss will be less able to simultaneously coordinate all the attributes in the task);

   c. use of information provided by corrections (older Ss will make better use of this information)

2. The experimental task will prove to be a valid and reliable method for identifying cognitive processes employed during problem solving.

Reporting Session

Saturday, April 21
9:00-10:00

INTELLECTUAL VARIABLES WHICH DISCRIMINATE GOOD AND POOR PROBLEM SOLVERS

Ruth Ann Meyer
Western Michigan University

Purpose

This exploratory study investigated intellectual variables associated with successful mathematical problem solving. Good and poor problem solvers were identified and then compared on a set of 19 variables. The study presents further analysis of Meyer's study, Mathematical Problem-Solving Performance and Intellectual Abilities of Fourth-Grade Children (1978).

Procedure

The subjects for Meyer's study were 179 fourth-grade children who were studying Developing Mathematical Processes (DMP) (Romberg, Harvey, Moser, Montgomery and Dana, 1974; Romberg, 1976) in Wisconsin, Illinois or New York. Twenty tests were administered to these children. Of these tests, 19 were "reference" tests for Verbal, Induction, Numerical, Memory, Perceptual Speed, Spatial Visualization, or Word Fluency ability. The remaining test was a mathematical problem-solving test constructed by Romberg and Wearne (1975). This problem-solving test was designed to yield three scores: a comprehension score, an application score, and a problem-solving score. To accomplish this, the test was composed of 19 items such as the following:

Example

There are 45 houses on Century Avenue. The houses are white, blue, yellow, green or gray. 18 of the houses are white. There are half as many blue houses as white houses. 15 of the houses are yellow. Only 1 house is gray. (Item Stem)

There are more blue houses than white houses. (Comprehension Question)

TRUE FALSE IMPOSSIBLE TO TELL

There are more blue houses than yellow houses. (Application Question)

TRUE FALSE IMPOSSIBLE TO TELL

How many of the houses are green? _____ (Problem-Solving Question)

The results for the three parts of the problem-solving test (Comprehension mean, 13.52; Application mean, 9.72; and Problem-Solving mean, 3.47) suggested the following questions:

1. If students understand the item stem of a problem, have mastered the necessary mathematics concepts and computational skills, why do they have difficulty solving the problem?

2. Do the reference tests for intellectual abilities administered to the sample provide any answers to the first question? Could these reference tests be used to discriminate between those students who performed well on all three parts of the Romberg-Wearne test and those who performed well on the comprehension and application parts but poorly on the problem-solving questions?

The following two groups were investigated in these exploratory analyses:

1. High-High-High Group (HHH)

   Those students whose scores were above the means for the comprehension and application questions and who had seven or more correct for the problem-solving questions were classified into this group. Frequencies scores were used for the seven or more correct criterion. Sixteen subjects satisfied the established criteria for this group.

2. High-High-Low Group (HHL)

   Those subjects whose scores were above the means for the comprehension and application scores and had fewer than seven correct for the problem-solving questions were classified into this group. Sixty subjects satisfied the criteria for this group.

Analysis

The hypothesis $\mu_{HHH} = \mu_{HHL}$ was tested for each of the 19 reference tests. Significant differences in the means were found at the .01 level for the five variables: Figure Matrix and Number Exclusion (Induction ability), Mathematics Computation (Numerical ability), and Vocabulary and Word Group Naming (Verbal Comprehension). Significant differences in the means were found also for six other intellectual variables at the .05 level.

Since so many significant differences were found, the investigator was interested in further characterizing the HHH and HHL groups. For this purpose, a stepwise regression procedure, in which the problem-solving score was the dependent variable and the 19 reference tests for intellectual variables were independent variables, was used.

Results

Figure Matrix had the largest F-ratio; consequently, it was entered first into the stepwise regression procedure. After Figure Matrix, Mathematics Computation accounted for the largest amount of variance in the problem-solving score; therefore, it was selected second by the procedure and was entered with Figure Matrix. Number Exclusion was the third variable entered. These three were the only variables which had significant F-values to enter into the procedure when Figure Matrix with the first entrant. The entire battery of 19 tests accounted for 42.7 percent of the variance of the problem-solving questions. Figure Matrix, Mathematics Computation, and Number Exclusion accounted for about three-fourths of this 42.7 percent.

Because stepwise regression procedures are highly dependent upon the variables entered first, and since Word Group Naming, Mathematics Computation, Number Exclusion and Vocabulary also had significant F-ratios at the .01 level for the HHH and HHL groups, each of these variables were entered as the first variable in the regression procedure. Regardless of the first entrant, Figure Matrix, Mathematics Computation, and Number Exclusion always emerged with significant F-values.

Conclusions

The results of this exploratory study suggest that Figure Matrix, Mathematics Computation, and Number Exclusion play important roles in successful problem solving. Nevertheless, the contributions of Word Group Naming and Vocabulary may not be ignored. Each of these two variables, when entered first into the regression procedure, also contributed significantly to the total variance of the problem-solving questions.

Generalizability of the results is restricted by unequal sample sizes, the difficulty of the problem-solving questions, and the high correlations between some of the intellectual variables. Yet, the significant differences of the means for 11 of the intellectual variables and the repeated emergence in the regression procedures Figure Matrix, Mathematics Computation, and Number Exclusion suggest that good and poor problem solvers do differ.

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THE RELATIONSHIP BETWEEN HEURISTIC HINTS AND THE SOLUTION PROCESSES USED BY STUDENTS CLASSIFIED AS CONCRETE OPERATIONAL OR FORMAL OPERATIONAL

Albert G. Watson
Columbia University

Purpose

The purpose of the study was to investigate the problem-solving processes used by students at the Piagetian concrete operational and formal operational stages of cognitive development and offered hints of a heuristic nature at appropriate points during solutions. Major questions were:

1. What is the relationship between the cognitive stage of a subject and the heuristic strategies that subject uses spontaneously when solving mathematical word problems?

2. What is the relationship between heuristic hints offered to subjects during their solutions to mathematical word problems and the strategies used on subsequent problems; what role does cognitive stage play in this relationship?

Kilpatrick (1969) has recommended that researchers who choose to investigate mathematical problem-solving processes undertake clinical studies of individual subjects because our ignorance of how students solve problems demands ex loratory work before larger scale experimental studies can be undertaken. Hatfield (1975) has pointed out that several features are absent from research done on heuristics: "Particularly missing are concerns for the mental operations which are . . . necessary to assimilate the heuristic precept to one's mental structure" (pp. 20-21). Webb (1975) raises two questions about the relationship between cognitive development and the use of heuristics:

1. Can strategies, such as those discussed by Polya, be used by students before they reach the formal operational stage?

2. At what age and in what form should the teaching of heuristics begin? (p. 103)

The relationship between cognitive development stage and the use of problem-solving processes is of interest to the problem-solving researcher because of the most distinctive feature differentiating formal thought from concrete thought—possibility as opposed to empirical reality (Inhelder and Piaget, 1958). According to Inhelder and Piaget (1958), the formal operational individual is committed to

possibilities and "begins to build 'systems' or 'theories' in the largest sense of the term" (p. 339). However, "concrete thought remains essentially attached to empirical reality" (p. 250) and the concrete child does not formulate hypotheses. Thus, it is quite possible that the concrete child is more limited than the formal child in ability to generalize and to recognize the structural relationships between problems, and to identify among the given terms of a problem the information maximally useful for its solution.

While there have been studies which investigated the relationships between cognitive stage and problem-solving processes (Days, 1978; Grady, 1976), these studies have been "one-shot" studies of individuals' problem-solving processes. The study herein described differs from these previous studies in that the number of subjects is smaller while the number of problems solved and the number of interviewing sessions with each subject are larger.

Wittrock (1973) suggests that learning is a generative process which involves "relating the structural organization to the learner's experience, and encouraging the learner to process the information" (p. 28). The generative aspects of mathematical learning was exploited in two ways. First, the experimenter directly interfered with his subjects' oral solutions to problems. This interference was similar in nature to the help Krutetskii (1976) gave to subjects. Krutetskii established a level of accomplishment attained by students during independent solutions to problems and also the level attainable with some help from the interviewer who provided hints, promptings, leading questions, and directions. Second, when subjects finished solutions, they were asked to look back at the solution in order to find if there was something in the nature of the problem or in the solution process which might be of assistance in solving future problems.

Procedure

The study was conducted in two phases. In the first phase subjects from a seventh grade class at an independent school in New York City were classified as concrete operational or formal operational according to performances on several Piagetian tasks which were administered in individual interviews with the subjects. After the students had been classified, four subjects from the formal group, four from the concrete group, and one from the transitional group were selected. Each subject was then interviewed five times at approximately weekly intervals, each session lasting no longer than 40 minutes. In these sessions subjects were asked to solve mathematical word problems while thinking aloud and all solutions were audio-recorded. Except during the first session, subjects' solution processes were interfered with by the experimenter. The interference consisted of heuristic hints given at appropriate stages of the solution processes. After completing solutions subjects were asked to look back at their solutions and to make judgments about the efficacy of the strategies that were employed and the hints that were offered. Looking back, in certain instances, included listening to the audio-recording of a solution.

Preliminary analysis of the first phase audio-recordings indicated that it might be fruitful to conduct a second phase differing from the first phase mainly in the number and frequency of interviewing sessions. In the second phase, four subjects—two each at the concrete and formal stages—were chosen and interviewed ten times each at twice weekly intervals. In other respects the second phase was similar to the first.

Analysis

All the audio-recorded protocols were transcribed and analyzed for evidence that subjects used certain heuristic strategies—draw a chart or figure, generalization, specialization, recall a related problem, use of equations, use of trial and error, working forward, and working backward. Selected protocols were analyzed in more complete detail using the system of analysis developed by the Combinations of Heuristics Working Group (Smith et al., 1977) and based on the system developed by Kilpatrick (1968).

Results

Results will be reported when the paper is presented.